Wānangatia te Putanga Tauira

National Monitoring Study of Student Achievement

Key Competencies 2017

REPORT 4:

Disciplinary meaning–making in mathematics and statistics

4

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National Monitoring Study of Student Achievement

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REPORT 4:
Disciplinary meaning–making in mathematics and statistics

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1. Introduction

This short report describes a retrospective analysis of assessment items used in the 2013 National Monitoring Study of Student Achievement (NMSSA) study of the mathematics and statistics learning area. The focus of the analysis was disciplinary meaning-making in mathematics. The report is one of a series of three investigations carried out by the NMSSA team focussing on student progression in the capability of meaning-making in different learning areas. The other two reports look at disciplinary meaning-making in English: viewing, and science.

The report begins by describing how 'disciplinary meaning-making' is defined and then outlines the particular meaning-making challenges in the mathematics and statistics learning area. Next we explain how we analysed the items to look for evidence of whether and how Year 4 and Year 8 students demonstrated aspects of the meaning-making capability as they responded to the assessment items. We conclude with some thoughts on how the findings might affect the treatment of meaning-making in the mathematics classroom.

2. Disciplinary meaning-making

Disciplinary meaning-making, along with critical inquiry and perspective-taking are the three capabilities that NMSSA uses to consider how well students have developed the key competencies described in the New Zealand Curriculum (NZC). Capabilities remix aspects of all the key competencies and weave them together with important knowledge and skills. They are then demonstrated in action. They show what learners are able and willing to do as a result of what they have learned.

The three capabilities that NMSSA has chosen to focus on result from a careful analysis of NMSSA tasks from across the different learning areas. Each capability can be seen in action across the curriculum and they are almost always needed in terms of successful task completion.

The capability of disciplinary meaning-making requires students to take the 'perspective' of the discipline (to think like a mathematician, scientist or literary critic for example). In turn, this implies being able to think critically and to manage the various types of texts that are appropriate to the discipline area.

Meaning-making is essential for accessing the ideas of others, as well as expressing understanding and ideas, and creating ideas. It is complex and multifaceted. The most direct key competency links are to *Using language, symbols and texts* and to *Thinking*.

Meaning-making in mathematics and statistics

The essence statement for mathematics and statistics in the NZC highlights how these two disciplines provide students with a meaning-making capability. According the NZC, mathematics and statistics "…equip students with effective means for investigating, interpreting, explaining and making sense of the world in which they live" (p.26). Part of this involves developing an understanding of the conventions that mathematical and statistical communities have developed to aid their thinking and to communicate ideas, data and reasoning. This includes, but is not limited to, the use of specific vocabulary, symbols, graphics and diagrams.

Meaning making conventions in mathematics[1] are tightly bound up with the mathematical ideas they represent and at times it is very difficult to separate the convention from the mathematics itself. Both, however, are essential to learners.

[1] For brevity, 'mathematics' will generally be used to refer to the mathematics and statistics learning area throughout this report.

3. How we analysed meaning-making in mathematics

We used two sources of information from the 2013 NMSSA study of mathematics and statistics to explore meaning making in mathematics. The first involved items from a paper-and-pencil assessment called the Knowledge and Application of Mathematical and Statistical Ideas (KAMSI) assessment. The second source was items from a computer-administered task called Making Meaning. Both sources are introduced below.

The KAMSI assessment

In the KAMSI assessment, about 2,300 students at each of Year 4 and Year 8 completed different selections of items, which were drawn from a bank of about 80 items that had been especially developed for the assessment. Most of the items were selected-response items. Some short-answer items were also used. Each student completed about 35 items.

An assessment linking design involving an additional sample of Year 6 students allowed NMSSA to develop a measurement scale that could be used to report achievement across the year levels (the KAMSI scale). All of the items used in the assessment were also able to be located on the scale according to their relative difficulty. The higher up the scale an item was located, the more difficult it was for students to answer the item correctly.

Figures 1 and 2 show the distribution of student achievement on the KAMSI scale in 2013 for Years 4 and 8 respectively. The scale itself is shown to the left of each figure and in both cases is numbered from 0 to 200 scale score units. As could be expected, on average, Year 8 students' levels of achievement tended to be located higher on the scale than those of Year 4 students. There was overlap between the achievement distributions for the year levels however.

The difference between the average scores at Year 4 and Year 8 was about 32 scale score units. This indicates that, on average, a year of instruction equates to about 8 units of progress on the scale.

Also shown in each figure, are a series of horizontal shaded lines that indicate the minimum scale scores (cut-scores) associated with achieving the curriculum objectives at each of curriculum levels 2 to 4. The cut scores were determined by a panel of curriculum experts through a systematic standard setting exercise.

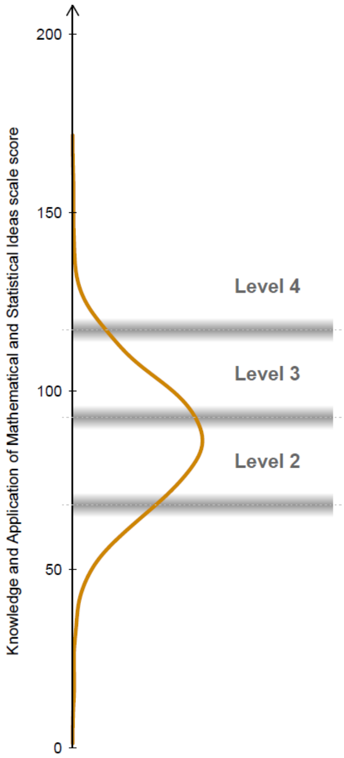

Figure 1. Distribution of Year 4 Knowledge and Application of Mathematical and Statistical Ideas scores aligned with curriculum levels.

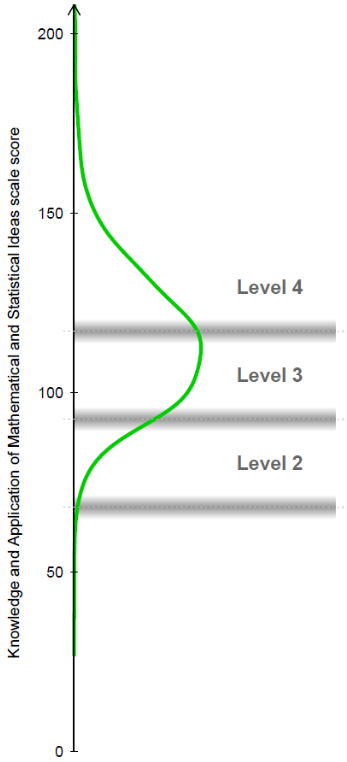

Figure 2. Distribution of Year 8 Knowledge and Application of Mathematical and Statistical Ideas scores aligned with curriculum levels.

To carry out the analysis we went back to the bank of KAMSI items and identified instances where we could see that students needed to draw on mathematical ways of meaning-making to complete the items. Several specific types of meaning making were evident across the bank. We selected four to investigate further:

- representing part-whole relationships through fractions
- using symbols to signify operations and relationships
- making sense of measurement scales
- making sense of graphs.

For each of the specific types of meaning-making we selected a number of items from the KAMSI assessment where that type of meaning-making was required. We developed a brief description of the disciplinary meaning-making demands of the selected items and then looked at where the items were located on the KAMSI scale. From this we developed four figures (figures 3, 4, 5 and 7) which displayed the descriptions associated with each item on the scale. These figures were used as a basis for describing patterns in the way the demands emerged and developed as the items became more difficult.

The Making Meaning task

The Making Meaning task provided a second source of information related to disciplinary meaning making in mathematics. This task was part of collection of tasks that looked at mathematical and statistical proficiencies. The tasks were administered separately to the KAMSI assessment and involved a sub-sample of the students in the NMSSA study (about 800 students at each year level). The Meaning Making task was focussed on students' understanding of a range of mathematical terms and symbols. For each term or symbol students were presented with a number of possible definitions and asked to select the one definition that best matched the term or symbol. In this part of the analysis we show each of the items that made up the task and report the percentage of students at Year 4 and Year 8 who were able to respond correctly.

4. Meaning-making in the KAMSI assessment

This section examines four different types of meaning-making associated with the mathematics and statistics learning area as evidenced through responses to items from the KAMSI assessment. The four types are:

- representing part-whole relationships through fractions
- using symbols to signify operations and relationships
- making sense of measurement scales
- making sense of graphs.

As described in the previous section, a figure is used in each case to display a selection of KAMSI items that involved the type of meaning making involved. In the figures, each box briefly describes an item in terms of its meaning-making demand. The blue arrow to the left of each item box is positioned to show the location of that item on the scale. Each item is located at the part of the scale where a student's overall level of achievement was associated with a 70 percent probability of answering the item correctly. In each figure the scales have been colour-coded to represent the curriculum level associated with achievement at each part of the scale.

Representing part-whole relationships through fractions

Using symbols to represent amounts is a fundamental idea in mathematics. Students need to develop key ideas about how amounts can be represented before they can use these representations to support and represent their thinking, as well as interpret the thinking of others.

Fairly early on in their learning, students are introduced to fractions as a way to represent a part of a whole. The number on the top of the fraction (the numerator) represents the number of parts we are focused on – how many, while the number at the bottom (the numerator), stands for the total number of parts needed to make up the whole. Developing a strong appreciation of how this notation can be used to represent a part-whole relationship, promotes students' ability to interpret problems, reason efficiently, and communicate effectively.

In figure 3 we have described three items, which were fundamentally about using fractions to represent a part-whole relationship. The positioning of the various items on the scale clearly illustrates the increasing demand associated with a growing understanding of the part-whole relationships signified using a fraction.

In item A the student has to identify which fraction represents the relationship between a part of a group of fruit and the whole group. Each part (3 apples and 1 lemon) is clearly shown. To work out what fraction of the fruit are lemons, students can count the number of lemons (1) and compare this to the total number of fruit (4).

In the second question however, simply counting the parts shown will not be sufficient. The student has to understand that the denominator represents the number of equal-sized parts that make up the whole. They will need to identify whether the part that is shaded can be repeated an even number of times to make the whole and, if so, how many repeats will be required.

The third question goes one step further. This time a rectangle has been divided into a number of regions of various sizes and more than one of these regions has been shaded. The student has to recognise which region can be used as a base unit that can be 'repeated' to both fill the shaded parts of the rectangle and to fill the whole rectangle. Success at this level indicates that students have a robust understanding of the part-whole relationship and how this is reflected in the symbols used to represent it.

Having a strong understanding of the part-whole relationship represented by a fraction allows students to reason their way through problems involving fractions. For instance, when asked to find 2/5 of 80, the student can know that they are working with a part that will divide into 80 five times (16) and that they are interested in two of these parts (32).

The scale locations for these items indicate that developing a robust understanding of the part-whole relationship takes time. The items are located over a range of about 50 scale score units. Results from the NMSSA mathematics and statistics study indicated that, on average, student achievement increased by about 8 scale score units per year. This would suggest that the difference in understanding between the first and last items represents the amount of progress associated with about 6 years of instruction.

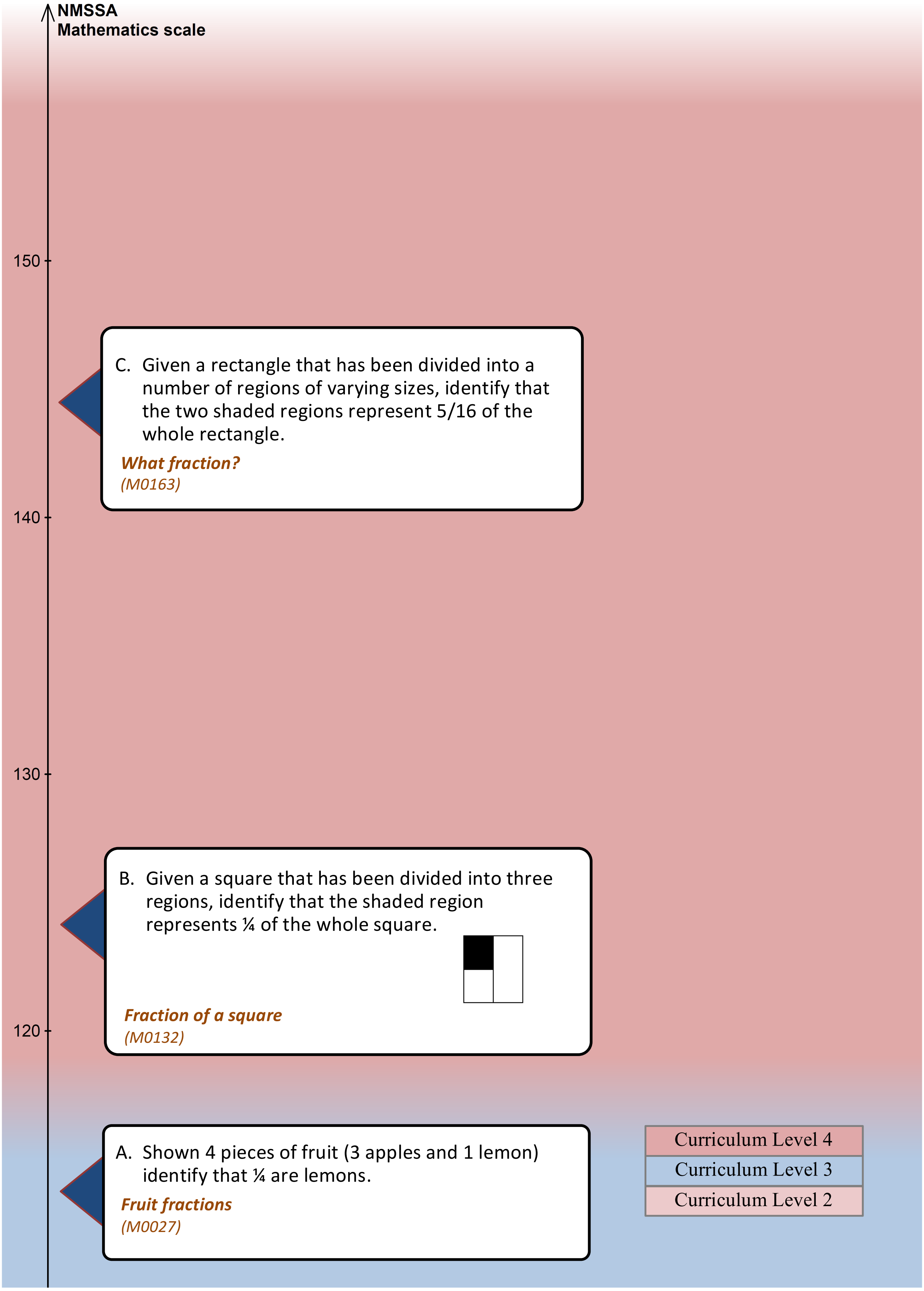

Figure 3. Figure 3 Representing part whole relationships using fractions

Using symbols to signify operations and express relationships

Mathematics uses a large range of symbols to signify operations and express relationships. As students develop as mathematicians they become more adept at using these symbols to think with and communicate ideas.

The items displayed in figure 4 require students to interpret symbols that signify operations and that work together to express mathematical relationships.

At the lower part of the scale shown (around 100 scale score units), students were typically able to identify that the multiplication symbol "×" corresponded with the arithmetic operation needed to solve a problem (item A). They were also able to identify the sequence of symbols and numbers that represented the mathematical operations required to solve a word problem (item B).

At higher levels students were able to recognise that a symbol, such as an empty box could be used to represent an 'unknown' and solve simple equations to find the value of the unknown component (items D and E). Students who were achieving slightly further up the scale again were able to interpret what the letter '*x*' was being used to represent in an equation (item F).

The most difficult item shown in figure 3 is item G. This item required students to find an unknown amount by solving an equation. The numbers and operations involved in the equation were straight forward – no more demanding than several of the items located much lower on the scale. However, in order to answer the question correctly, students had to consider which operation (multiplication or addition) should be done first. This required knowledge of the order of operations and appears to have been a 'stumbling block' for many students.

The order that mathematical operations are carried out in, rests on a set of rules or conventions. It is not as simple as just carrying out the operations in the order they occur. Failing to adhere to the rules will most often result in different values for the same mathematical expression.

Students also found it difficult to solve a relatively simple equation where an appropriate understanding of the equals ("=") sign was critical (item E). When responding to this item ($3 + \square = 7 + 2$) students often wrote a '4' in the box. This indicates that students are often reading the equals sign as a direction to carry out the operation that precedes it, rather than a symbol that signifies that the left side of the equation is equal in value to the right side of the equation. This misconception involving the equals sign is well documented in mathematics education research and often persists well into secondary school.

The responses to these questions indicate that by Year 8 many students have yet to develop a strong understanding of the order of operations and the meaning of the equals sign. Both of these are critical to success in algebra. Given the generally straight forward nature of the numbers and operations involved in the items that involved these concepts it is probable that many more students would have achieved success on the items if they had a stronger grasp of these ideas.

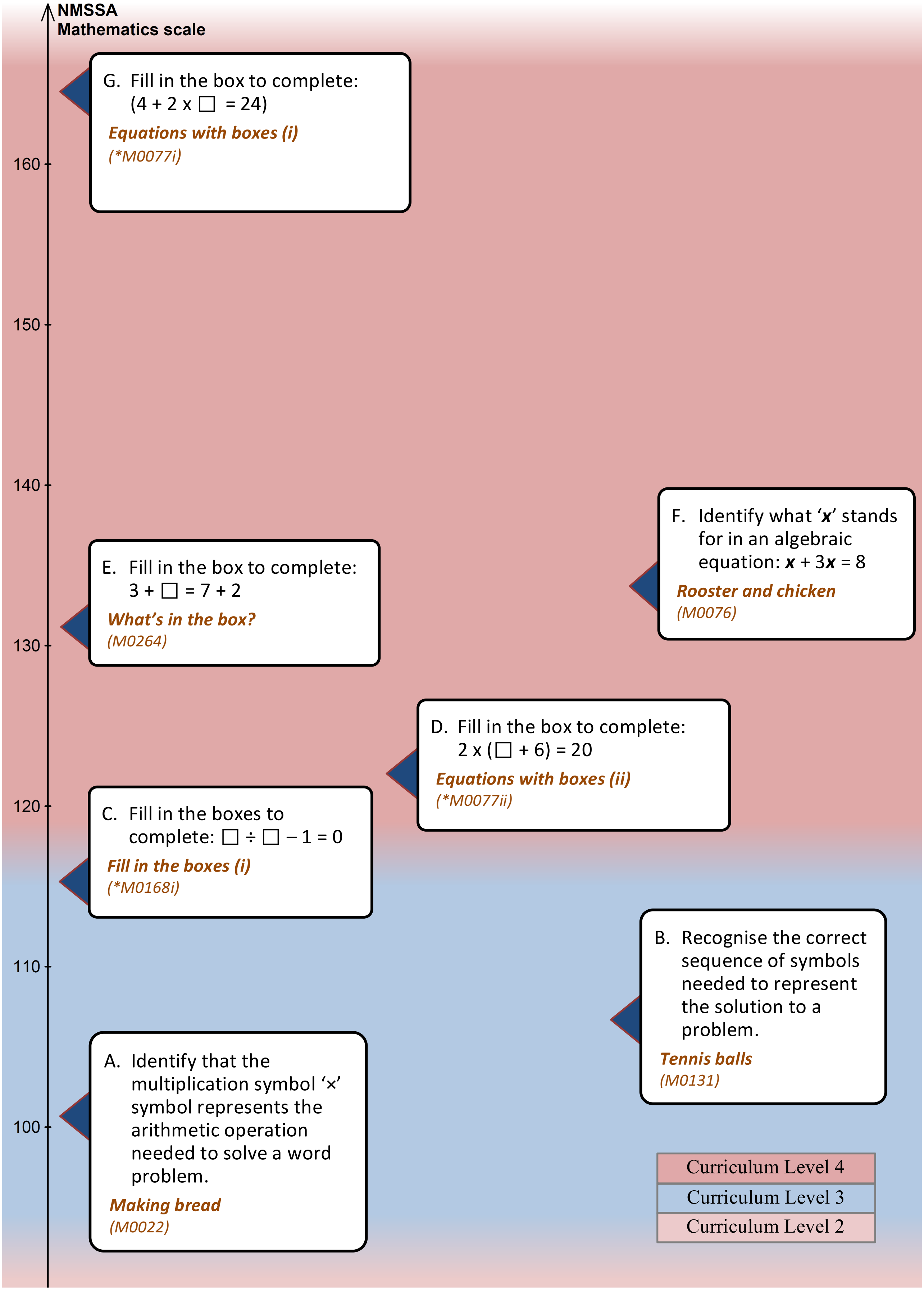

Figure 4. Using symbols to signify operations and express relationships

Making sense of measurement scales

Making a measurement often involves reading a scale. Scales are often presented in the form of number lines. To make meaning from a scale, students have to be able to relate the numbers shown on the scale to the 'tick' marks or divisions that form the scale. This includes being able to use the divisions to partition the unit of measurement into sub-units often involving fractions and decimals.

Figure 4 shows a series of items included in the KAMSI assessment that required students to read a measurement scale.

Items A, C and E all involved making sense of an unnumbered point on a scale. For instance, in item C, students had to identify the measurement associated with an unnumbered point halfway between 17 and 18.

Students found it easier to determine the correct measurement when the unnumbered point on a scale represented a whole number half-way between two other whole numbers (item A) than when a fraction or decimal was required to name the point (items C and E). The amount of progress represented by the difference in scale locations for items A and E is equivalent to about two and a half years of instruction.

Items B and D contrast the demands involved in using non-standard units to measure an object (item B) with the demands involved when using a standard scale (item D). In item B, the easier of the two items, students were presented with a picture of a car lined up against a row of pegs and asked to use the pegs to measure the length of the car. The car stretched from the start of the third peg to half-way along the sixth peg. In item D, the more difficult item, a paua shell was lined up against a ruler marked out in centimetres. One end of the shell was positioned at the 2 cm mark and the other end at the 6 cm mark. Students were asked what the length of the shell was in centimetres. Many of the students who answered item D incorrectly selected 6 cm as the answer. This suggested that they read the scale without taking into account the starting point of the measurement. The difference in scale locations between items B and D represents about one and a half years of 'progress'.

Measurement makes it possible to use numbers to describe the world around us. Learning how to measure involves developing an understanding of what counts as a measure. This includes learning the conventions involved in making a measurement and developing an understanding of how to read a scale. The responses to these items indicate that this is a process that takes time and that is deeply connected to students' developing number sense. The responses also indicate that students need opportunities to make sense of different types of scales that involve different graduations.

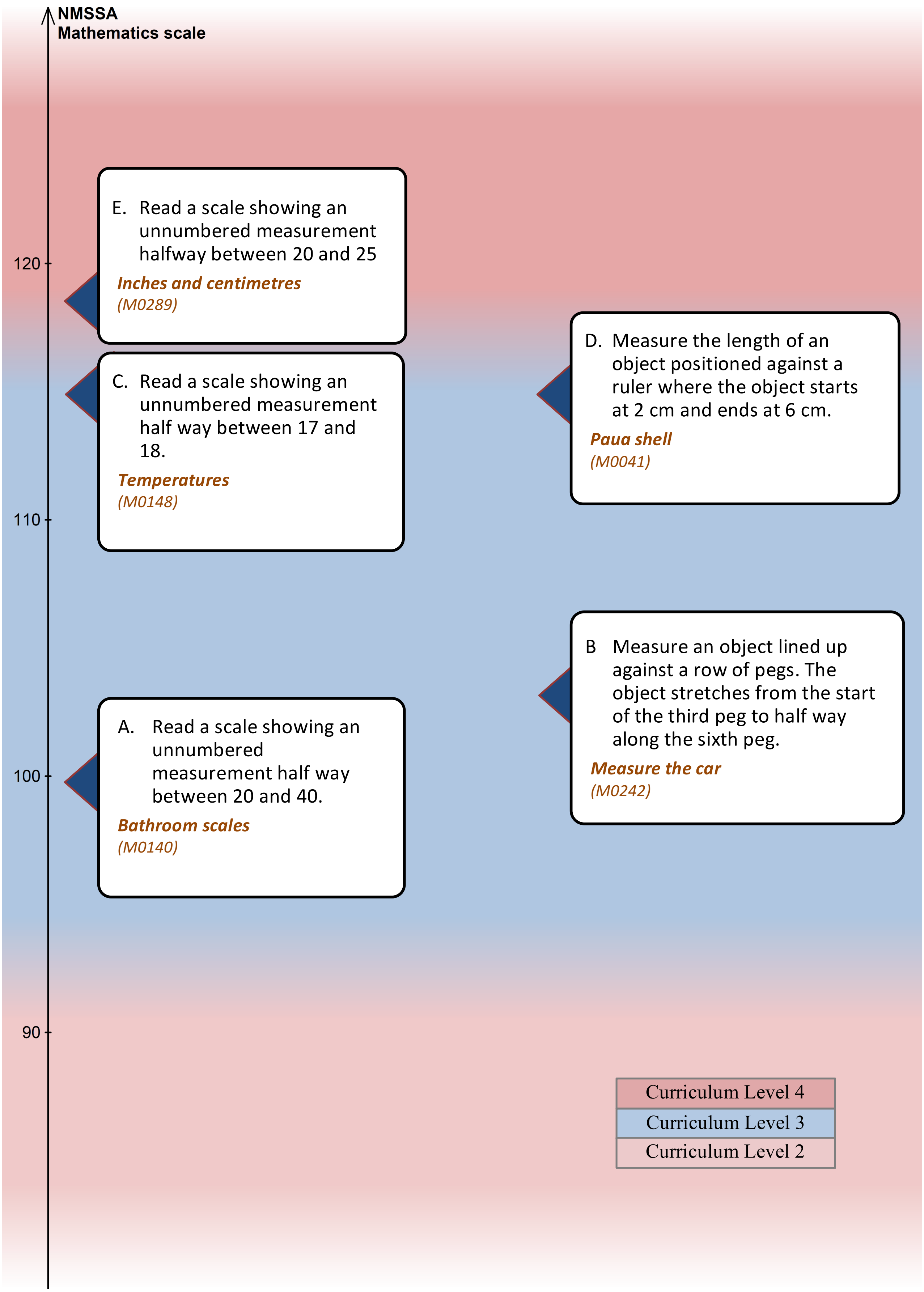

Figure 5. Making sense of measurement scales

Making sense of graphs

Graphs provide a powerful tool to describe and summarise data. They can also be used as a basis for reasoning and to make predictions. Learning how to use graphs depend on internalizing a number of graphing conventions that standardise how we make sense of data presentations.

Figure 7 describes a collection of items from the KAMSI assessment that were focused on students' ability to read and interpret graphs.

The items at the lower end of the scale generally required students to show they could make basic sense of data presented in a common graphing format. For instance, item A asked students to identify which of four bar charts matched a given tally chart. Item C asked students to identify which statement about the amounts represented by different sectors on a pie chart was true.

Most of the items further up the scale (items D, E, F, and G) either expected students to be able to recognise statements that accurately summarised the data distribution as a whole (for instance, accurately described a general trend), or involved having to use an element of proportional reasoning in order to interpret the data display. On average, these items were about 30 scale score units higher than the earlier items.

Students could get 'thrown' however, when making what appeared to be a simple reading from a graph. In item H, the most difficult item shown in the figure, students had to identify which of four simple statements about a dot plot was true. Figure 6, below, shows the dot plot used in the item. The plot records the number of sisters for each of the students in a class. These range from 0 sisters to 4 sisters. Responses to the item indicated that students found it difficult to interpret the relationship between the dots and the values or categories on the x-axis they were being used to represent. Many students presumed that they could simply count all the dots shown (21 dots) to work out the total number of sisters that were represented[2]. This didn't take into account that the number of sisters represented by each dot varied from 0 to 5 depending on which column the dot was placed in. Instead of 21 sisters, there were actually 27 represented in the data set.

Counting the dots would have worked if the categories themselves were not numbers. For instance, if the categories had stood for different car colours and the question had asked how many cars were represented. It is possible that students are most often meeting dot plots where the categories represent different things, and need more exposure to plots where the categories represent different numbers of the same thing.

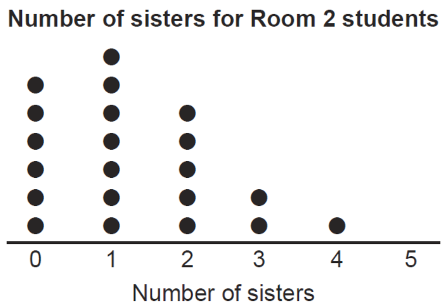

Figure 6. The dot-plot used in item H

[2] Counting all the dots would tell you how many people were in Room 2.

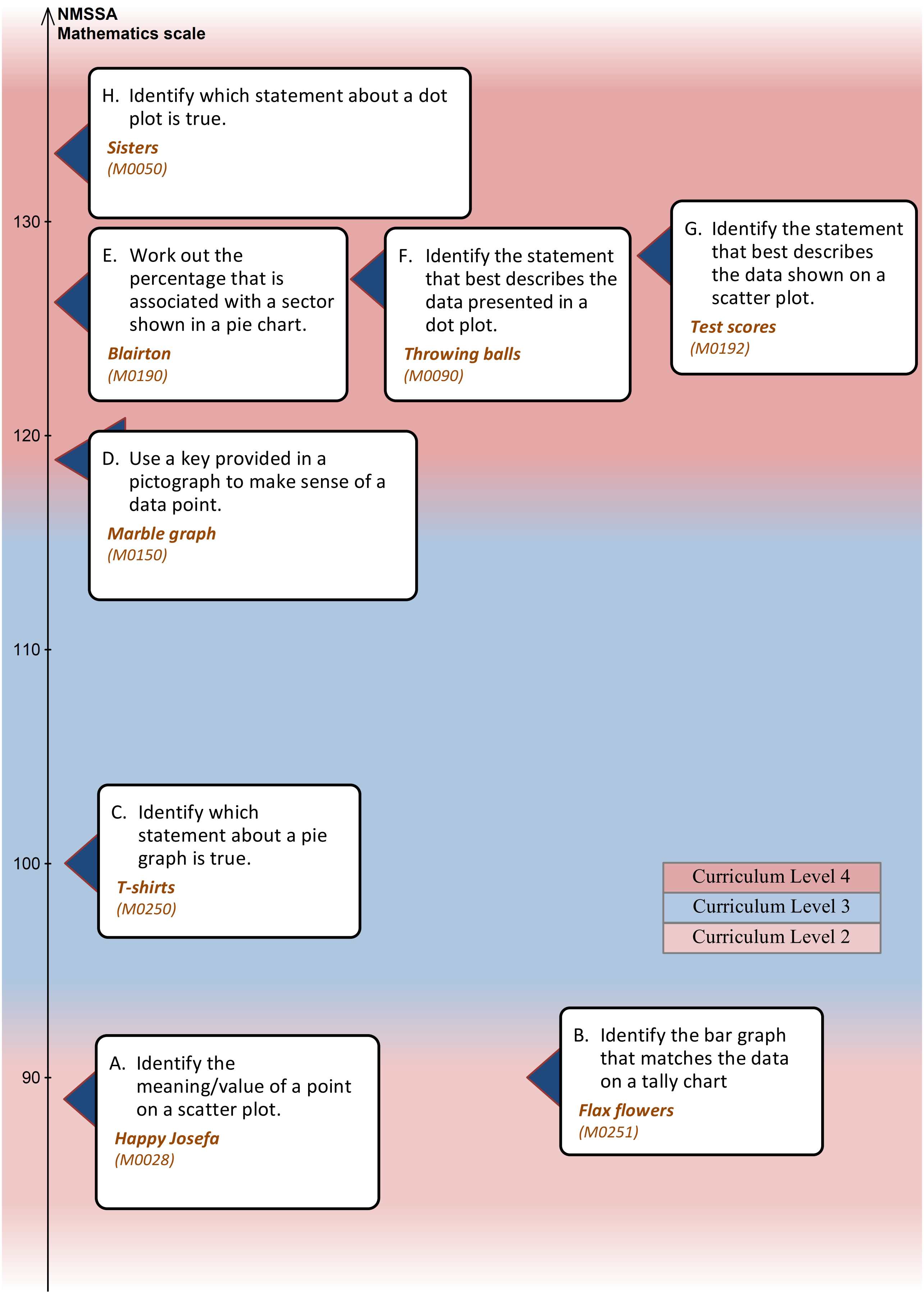

Figure 7. Making sense of graphs

5. The Maths Meaning task

The Maths Meaning task was made up of fifteen separate questions. The figure below presents screenshots of the questions in the task. Some questions were asked of students in both year levels and some of students in Year 8 only.

On the computer you will see some words and symbols that are used in mathematics.

Click on the box which best describes the word or symbol shown.

Screen shots from Maths Meaning task

Figure 8. Screen shots from the Maths Meaning task

Figures 9 and 10 show how students performed on the questions. Figure 9 focuses on symbols and terms used in the number and algebra strand and Figure 10 on those used in the geometry and measurement, and statistics strands. Year 8 students consistently out-performed Year 4 students. However, there were large proportions of students at both year levels who were unable to select an appropriate definition for many of the terms.

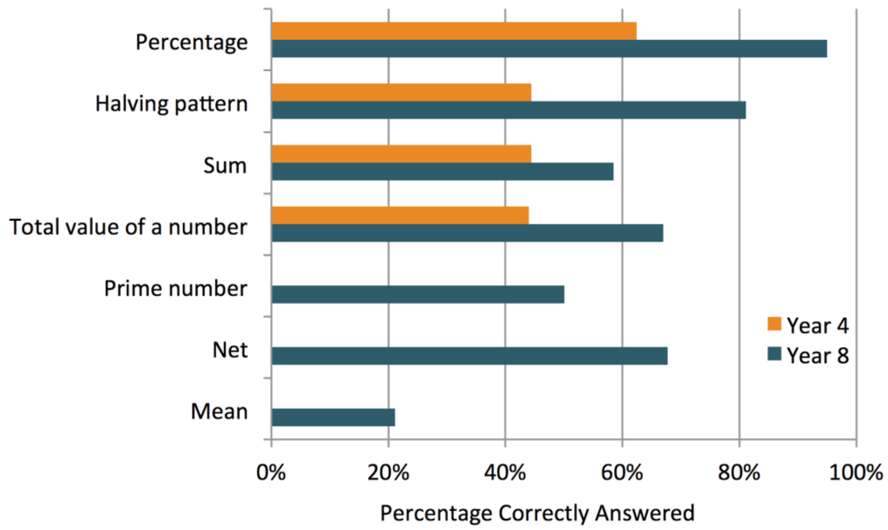

Figure 9. Understanding terminology and representations related to Number and Algebra: Percentage of students answering each question correctly by year

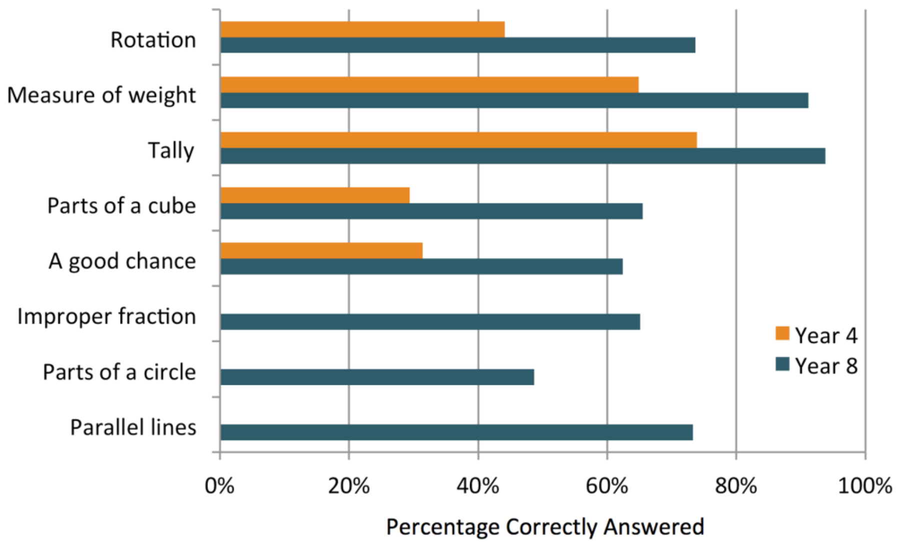

Figure 10. Understanding terminology and representations related to Statistics, and Geometry and Measurement: Percentage of students answering each question correctly by year

6. Final thoughts

The mathematics and statistics learning area is replete with symbols, terms and conventions that support our ability to think about and use mathematics, including our ability to communicate and use those ideas with others. The analysis reported here indicates that learning to make meaning both with and of mathematical symbols, terms and conventions takes time and that sometimes these mathematical meaning-making tools can become ‘stumbling blocks’ in their own right.

The analysis has not been able to gauge the extent to which students have opportunities to learn and interact with different aspects of meaning-making in mathematics. It would suggest, however, that there is plenty of room for students to develop a broader repertoire of meaning-making skills, especially at Year 8.

Many of these ideas, for instance, ideas about the meaning of the equals sign or the order-of-operations, can be taken for granted in the classroom. This can put students at a disadvantage, especially when the ideas become important tools for making sense of new mathematical ideas and contexts. The same ideas, however, can also be used as the subject of rich mathematical activity in the classroom where their meaning is discussed and explored. Overall, it would appear that more purposeful interaction with meaning-making ideas in mathematics and statistics would be beneficial to students as they develop the ability to use mathematics to make sense of the world.