NMSSA
Wānangatia te Putanga Tauira
National Monitoring Study of Student Achievement

INSIGHTS for Teachers

NMSSA Mathematics and Statistics 2022

Thinking about fractions

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Contents

Introduction

The following question has been used to monitor national achievement in mathematics and statistics at Year 8 since 1997. Before you read on, take some time to think about the question. How would you solve the question? How difficult do you think it might be for a typical Year 8 student? What answers might students give and how might they explain their thinking? How might performance on the question have changed since 1997?

$$\frac{1}{2} + \frac{1}{4} = ?$$

In 1997, 45% of the Year 8 students who were asked the question answered it correctly. Table 1 shows the percentage correct in the years it has been asked since then. As can be seen, there has been a general downward trend in the percentage of students answering correctly.[1]

TABLE 1: **Percentage correct for ½ + ¼ = ?, at Year 8**

Year	Percentage correct at Year 8
1997	45
2001	44
2005	42
2018	33
2022	32

There could be many explanations for the downward trend. One of these might be that over the years curriculum changes have reduced the amount of emphasis put on fractions generally and on adding fractions presented as equations, more specifically. Whatever the reason, when the NMSSA team was preparing for the 2022 mathematics and statistics study, we wondered why so many students were having difficulty with this particular question. On the face of it, the question looks relatively straightforward. Although it involves adding fractions with different denominators, ½ and ¼ are fractions students often come across. We felt that even when students didn't have much experience adding fractions, many of them should still be able to visualise the fractions in the equation, and problem solve their way to an answer. What was stopping students from activating their problem-solving skills? Were the students finding it hard to make sense of a problem that was presented out of context using symbols? Did students find it difficult to work with fractions? Were they answering too quickly?

The team decided that the 2022 NMSSA study should take a more in-depth look at fractions. This included carefully choosing a series of tasks to illuminate how students were thinking about fractions.

This booklet describes some of the thinking we noticed when we looked at the students' responses to the fractions tasks. After a short discussion of fractions, each part of the booklet looks at a task included in the 2022 study and unpacks a range of students' responses. Alongside, we have included some questions to think about and some space to jot down any thoughts or answers. At the end of the booklet, there are some suggestions for possible follow-up activities and links to some resources that might be helpful. Table 2 names each of the tasks covered and the focus idea associated with each one.

The booklet has been written primarily for teachers. If you are a teacher, we hope the booklet will stimulate some thoughts about your mathematics programmes and help generate some ideas you can use in the classroom and share with colleagues.

1 The question was first asked in 1997 as part of the National Educational Monitoring Project (NEMP). It was also used by NEMP in 2001, and 2005. NMSSA used the question in 2018 and 2022.

TABLE 2: **The tasks and the focus ideas**

Task name	Focus idea
Shaded Circle	Fractions involve equivalent parts
Chocolate Bar	Fractions are focused on a whole
Fractions	Fractions can be represented in multiple ways
Coloured Counters	Fractions can be compared with each other
Closest to $\frac{3}{5}$	Fractions are numbers

What are fractions and why are they important?

Fractions can be interpreted in different ways. The most common interpretation is as a way to quantify a part of a whole (a part–whole interpretation).[2] Fractions do this by showing how many parts we have of a 'given size'.[3] For instance, $\frac{3}{4}$ represents the idea that we have 3 parts, each of size $\frac{1}{4}$. Note that a fraction can represent more than one whole. For instance, $\frac{11}{4}$ means we have 11 parts, each of size $\frac{1}{4}$.

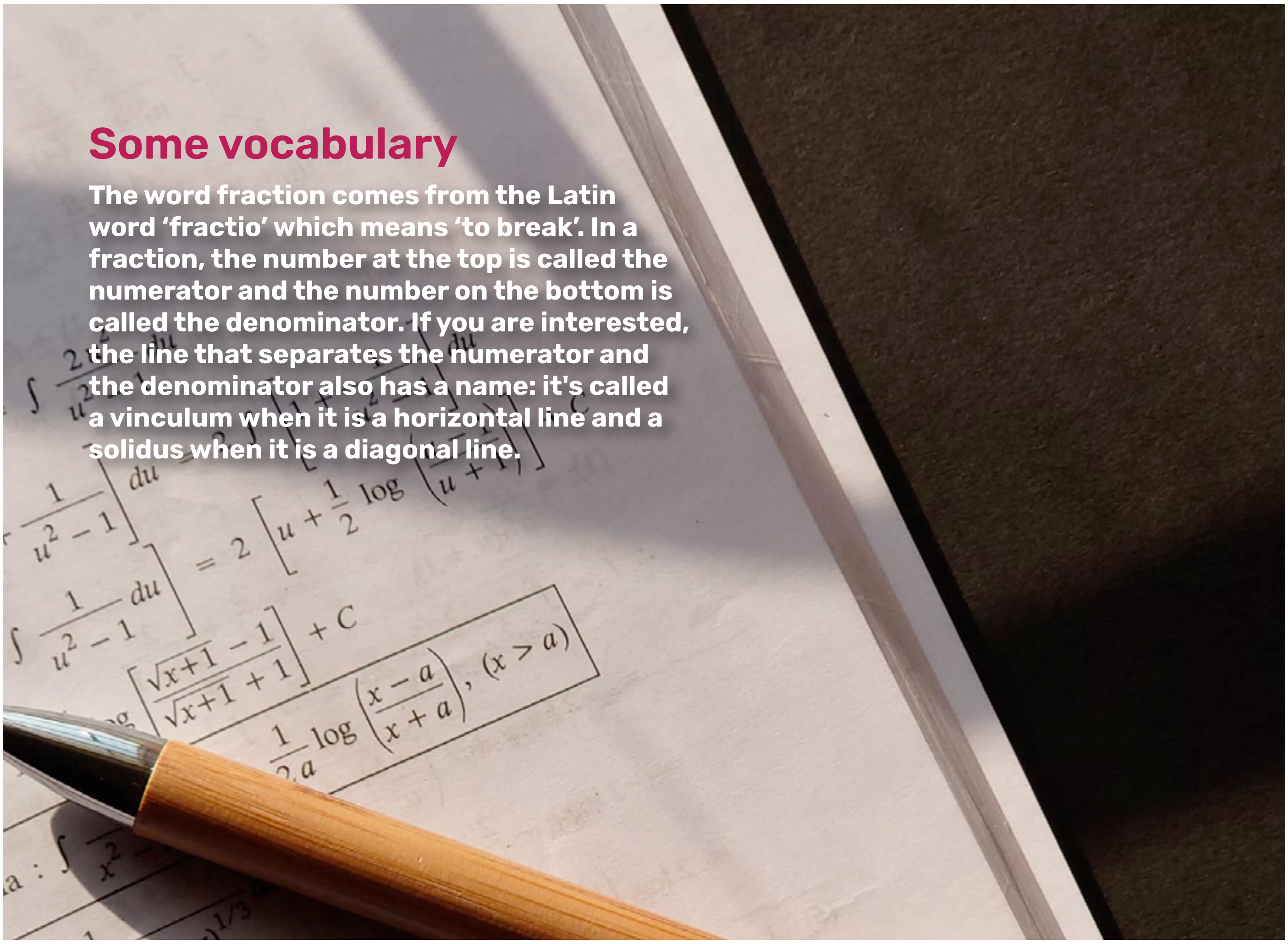
Fractions can have other interpretations; for instance, they can be understood as quotients, measures, ratios, and operators (scaling factors). A fundamental understanding when working with these interpretations in primary and intermediate school mathematics is that fractions are numbers that can be located on a number line and compared with other numbers, as well as added, subtracted, divided, and multiplied.

Understanding fractions is important. We often use fractions in our daily lives when doing things like following recipes, telling the time, measuring, and working with money and bank accounts. At school, fractions underpin many of the mathematical ideas our students will be faced with as they progress in mathematics. For instance: decimals, percentages, rates, and gradient all involve fractions. Fluency with fractions, including how to represent and work with them symbolically, is also a foundation for algebra. Algebra has been identified as a 'gateway topic' in mathematics. Success in algebra opens doors to all sorts of opportunities for further learning and careers.

Learning about fractions can be challenging. They represent a big conceptual jump from thinking about numbers as things we use to count with, to thinking about new sorts of numbers that exist between the counting numbers and that express a relationship between a part and a whole. This relationship, moreover, involves multiplication, which is a big jump from the world of adding and subtracting that students have often just begun to master when they meet fractions for the first time at school. These conceptual jumps can be hard

2 The part–whole interpretation extends to a 'part of a set' interpretation. For example, two chairs as part of a set of 10 chairs can be represented by $\frac{2}{10}$.

3 Note that 'given size' refers to an attribute of the whole or set that is equivalent across all the parts or subsets that make up the whole. This means that the parts might have different shapes or lengths but some attribute of each part, such as its area or volume or the number in each subset, is the same.



Some vocabulary

The word fraction comes from the Latin word 'fractio' which means 'to break'. In a fraction, the number at the top is called the numerator and the number on the bottom is called the denominator. If you are interested, the line that separates the numerator and the denominator also has a name: it's called a vinculum when it is a horizontal line and a solidus when it is a diagonal line.

to make, and some students can lose confidence or develop a negative perspective about mathematics when fractions are introduced into learning.[4]

Overcoming the challenges presented by fractions takes time and good teaching. This includes giving students multiple opportunities to work with problems involving fractions. It also involves moving beyond just concentrating on correct answers, to noticing, recognising, and responding to how students are thinking about and representing fractions. In the past, learning about fractions has often been associated with the memorisation and application of rules and procedures (such as using cross multiplication to add fractions). These methods can be useful, but when they replace, instead of extend, conceptual understandings, they can work against attempts to educate our students to be problem solvers and sense makers.

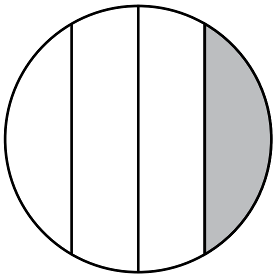
4 Note that students' understandings of whole numbers and fractions develop together in an integrated, concurrent manner. They are interdependent.

1. The Shaded Circle task

Students in Year 4 and Year 8 were presented with the Shaded Circle task shown below. The students completed the task independently.

FIGURE 1: **The Shaded Circle task**

Here is a shaded circle.



1. a) Is $\frac{1}{4}$ of this circle shaded? ☐ Yes ☐ No *(Tick response)*

   b) Explain why you think this. You can use words and pictures.

Questions

What percentage of students at Year 4 and Year 8 do you think answered: 'Yes'?

Year 4 ____ %

Year 8 ____ %

Why might a student think that the circle is ¼ shaded?

What is this task about?

A key idea when working with a part–whole interpretation of fractions is that the whole has been divided into a number of **equivalent** parts. In this task, students needed to be sure that what they were defining as the parts represented an equivalent part of the circle in terms of area. This circle has been carefully divided into four parts separated by lines. Each part has the same width; however, they do **not** all have the same area. In terms of area, therefore, the shaded part does not represent ¼ of the circle.

This task required students to look closely at the picture. A single glance might not be enough to recognise that the area of each part is not the same.

How did the students do?

Most students, at both year levels indicated that the circle **was** ¼ shaded (see Table 3).

TABLE 3: **Student responses to the Shaded Circle task**

Response	Year 4 (%)	Year 8 (%)
Yes—the circle is ¼ shaded	86	81
No—the circle is not ¼ shaded	14	19

How did the students explain their answers?

The students' explanations for their answers could be divided into three groups: those who did not mention the need for equally sized parts; those who did recognise the need for equally sized parts but who stated that the parts were equal; and those who recognised that the parts were unequal and therefore the circle was not ¼ shaded (see Table 4). About 3% of Year 4 student responses and 13% of Year 8 responses were in this last category.

TABLE 4: **Students' explanations for their answers**

Student response	Year 4 (%)	Year 8 (%)
Does not mention the need for four equal parts.	90	67
Recognises the need for four equal parts but argues the circle is ¼ shaded.	7	20
Explains that the parts are **not** equal and therefore the circle is not ¼ shaded.	3	13

The students who indicated that the shaded part did **not** represent a quarter of the circle, argued that the four parts shown did not have the same area. Some drew diagrams to show how a circle could be divided into four parts so that the area of each part was the equal.

TABLE 4: **Examples of students' explanations for why the circle is *not* ¼ shaded**

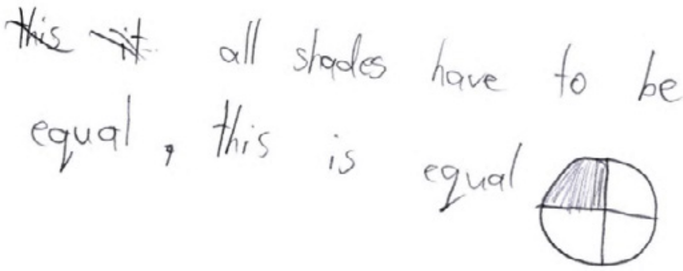
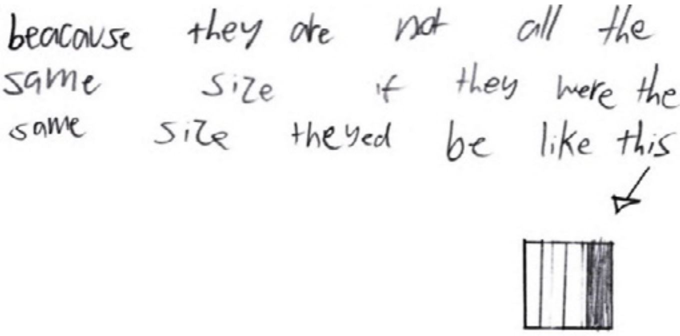
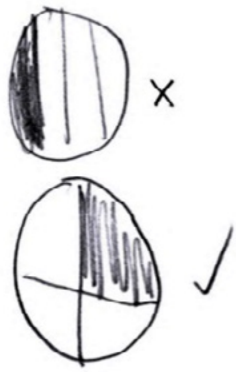
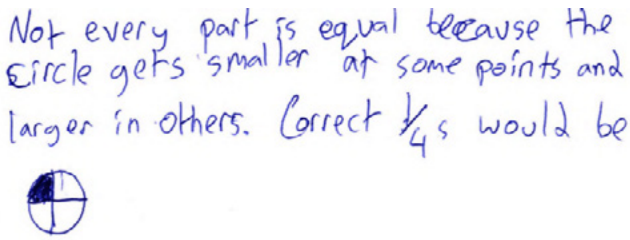
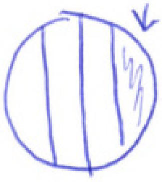
	This student argues that the shaded pieces are not equal and then demonstrates how a circle can be divided into 4 pieces with equal areas.
	This student also recognises the need for the parts to be equal in area. They have used a square rather than a circle to represent quarters based on equal areas.
	This student makes the representations do all the explanatory work. Note that, although vertical lines can easily be used to partition rectangular regions, they are not easily applied to circles.
	This student attempts to explain why the areas of the parts in the original circle are not all the same. They also provide a picture of a circle divided into quarters based on area.

Table 5 provides some examples of responses from students who argued that the circle was divided into 4 equal parts. These students could be looking at the evenness of the spacing between the cuts that separate the parts rather that the area taken up by each part. Alternatively, they might not have noticed that the area of each part is not the same.

TABLE 5: **Examples of students' explanations claiming that the parts in a fraction have to be equal but still indicating the circle is ¼ shaded**

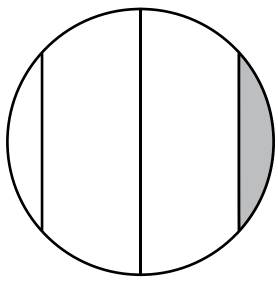
The circle is evenly cut in 4 and 1 bit is shaded.	Note the emphasis on evenly spaced cuts rather than parts with equal areas. It is true that the width between each cut is the same.
Because there are 4 peices and 1 is shaded in, to make 1/4, and the peices are equal.	This response and the one below defines ¼ in terms of 1 of 4 equal pieces. However, they don't define what is equal about each part.
 1/4 and it is shaded because it is splet in to 4 even Peice. So it 1 of the qurtes is shad so it is 1/4	

An extension to the Shaded Circle task

The students who completed the Shaded Circle task were given the follow-up question below (Figure 2).

FIGURE 2: **Follow up question for the Shaded Circle task**

Here is another shaded circle.



2. Is $\frac{1}{4}$ of this circle shaded? ☐ Yes ☐ No *(Tick response)*

The circle in the follow-up question has been drawn to exaggerate the difference between the parts—the area of each part is obviously different and the spacing of the parts is uneven. Table 6 shows how students responded to the follow-up question. As can be seen, compared to the first circle, a greater percentage of students indicated that the shaded part does not represent ¼. It would be useful to ask the students to explain their answers—are those who say that this is not ¼, thinking about the uneven spacing of the cuts, or are they looking at the differences in the area of each part?

TABLE 6: **Student responses to the second part of the shaded circles task**

Response	Year 4 (%)	Year 8 (%)
Yes—the circle is ¼ shaded	49	31
No—the circle is not ¼ shaded	51	69

2. The Chocolate Bar task

In the Chocolate Bar task, students in Year 4 and Year 8 were presented with a picture of part of a chocolate bar. They were asked to draw what the whole bar would look like. At Year 8, students were shown $\frac{4}{6}$ of the chocolate bar. At Year 4, they were shown $\frac{2}{3}$. The Year 8 students were also asked to explain their answers. Students completed the task as an independent activity.

FIGURE 3: **The Chocolate Bar task at Year 4 and Year 8**

Year 4	Year 8
1. This is $\frac{2}{3}$ of a chocolate bar. Add to the picture to show what the whole chocolate bar would look like. 	1. This is $\frac{4}{6}$ of a chocolate bar. Add to the picture to show what the whole chocolate bar would look like.  2. Explain how you worked out how to draw the whole bar.

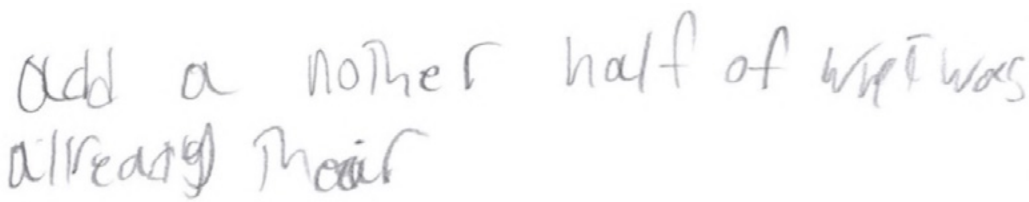
Questions

1. What percentages of students do you think were able to draw the whole bar at Year 4 and Year 8?

Year 4 [%]

Year 8 [%]

2. Have a look at this student's explanation for the Year 8 task. What understanding are they showing about fractions?



"Add another half of what was already there."

What is this task about?

The Chocolate Bar task is an example of a 'given a part, find the whole' question. The students are being asked to work their way from a known part to the original whole. To do this, the students must recognise how the number of shown parts relates to the total number of parts needed to reconstruct the whole. For instance, if the part shown is $\frac{2}{3}$, then one more $\frac{1}{3}$ part will be needed to make the whole. The Year 8 version of the task is arguably more demanding than the Year 4 version. Year 8 students need to recognise that four $\frac{1}{6}$ parts are shown and that they require 2 more $\frac{1}{6}$ parts to make the whole bar.

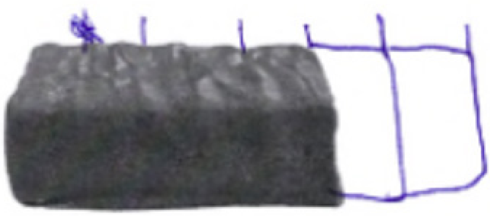
How did they do?

As might be expected, Year 8 students were more successful than those in Year 4, with 67% of Year 8 students drawing an accurate representation of the whole bar (see Table 7 and Table 8).

TABLE 7: **Student responses to the chocolate bar task**

Response	Year 4 (%)	Year 8 (%)
The student accurately represents the whole bar.	44	67

TABLE 8: **Examples of student responses to the Chocolate Bar task**

	The student has clearly added on 2 pieces. The pieces are drawn in proportion to the rest of the bar (they each represent a $\frac{1}{6}$ piece).
	The student had divided the part shown into 4 parts and added on 2 more. The divisions are not drawn accurately.

How did the students explain their answers?

The students in Year 8 were also asked to explain how they worked out how to draw the whole bar. About half of the Year 8 students were able to provide a convincing explanation. Table 9 shows some of the ways students explained their answers.

TABLE 9: **Examples of explanations at Year 8**

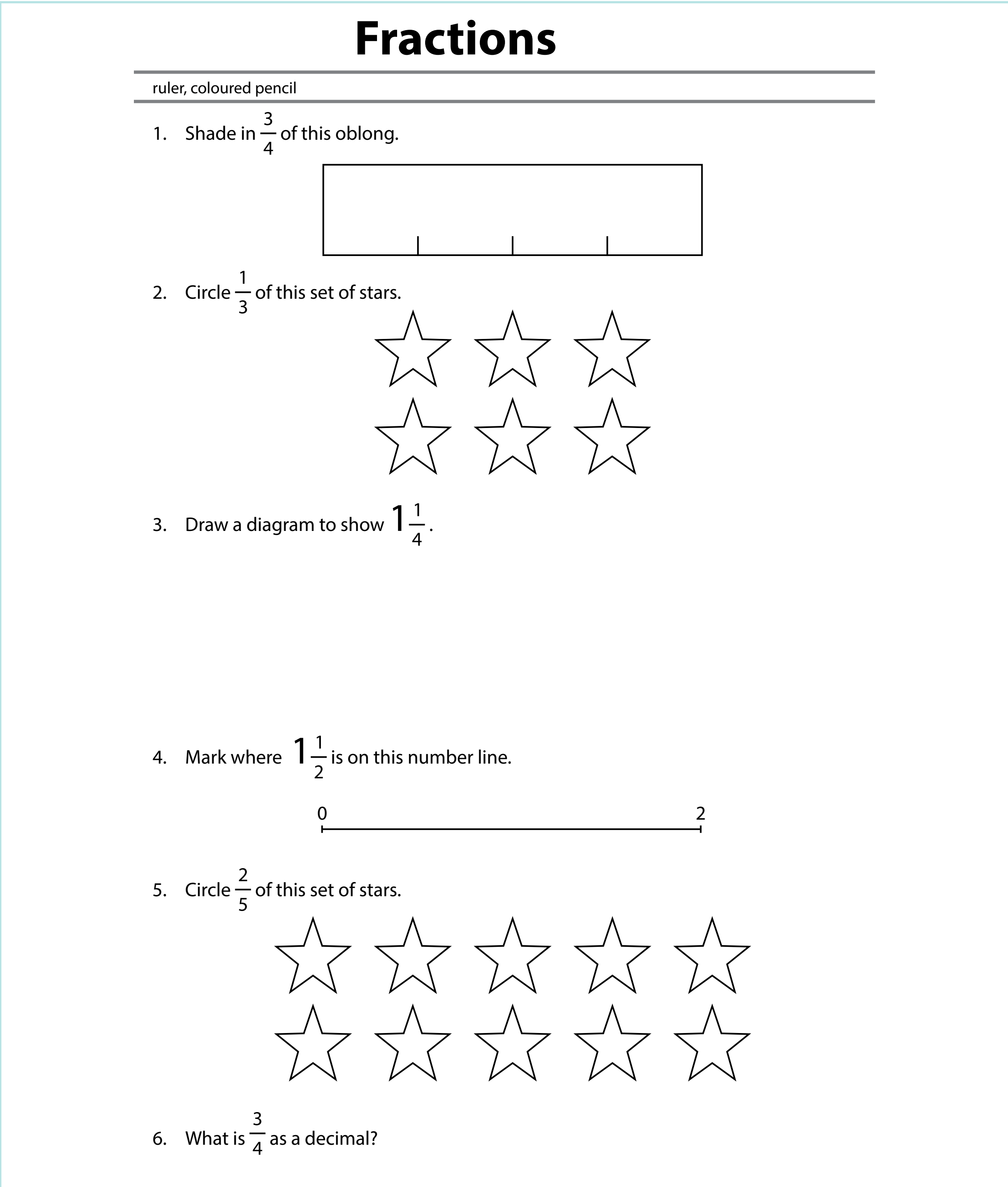
$\frac{4}{6} = \frac{2}{3}$ The chocolate bar is $\frac{2}{3}$ full and $\frac{1}{3}$ missing, so I drew one third of it.	This student has converted $\frac{4}{6}$ to $\frac{2}{3}$ before explaining how they identified the missing part.
there was four centimeters of chocolate so there had to be two centimeters less	This student has used ideas about measurement to find the missing piece.
I evenly drawed lines on the picture to show how much $\frac{4}{6}$ of a chocolate bare is, then i drew the ~~other~~ $\frac{2}{6}$ bit of the bar and then added it together.	This student has emphasised the need for evenness and how the markings they have used show the fractions they are working with.
add a nother half of what was already their	The student appears to realise that half of what is already there will represent $\frac{2}{6}$ of the bar ($\frac{1}{2}$ of $\frac{4}{6}$). Adding $\frac{2}{6}$ to the original $\frac{4}{6}$ will recreate the whole bar.

The students who struggled to reconstruct the whole bar provided a wide range of explanations. Some of them described not knowing what to do, while others explained they had taken a guess. A small proportion described using symmetry or their knowledge of chocolate (e.g., "I know what a chocolate bar looks like") to complete their drawing.

3. The Fractions task

The Fractions task asked Year 4 and Year 8 students to represent fractions using a range of visual models (area, set, and linear models) and as a decimal. The questions involved varying complexity—both in terms of the fractions they involved, and the representation used.

FIGURE 4: **The Fractions task**



Fractions

ruler, coloured pencil

1. Shade in $\frac{3}{4}$ of this oblong.
2. Circle $\frac{1}{3}$ of this set of stars.
3. Draw a diagram to show $1\frac{1}{4}$.
4. Mark where $1\frac{1}{2}$ is on this number line.

   0 — 2

5. Circle $\frac{2}{5}$ of this set of stars.
6. What is $\frac{3}{4}$ as a decimal?

Question

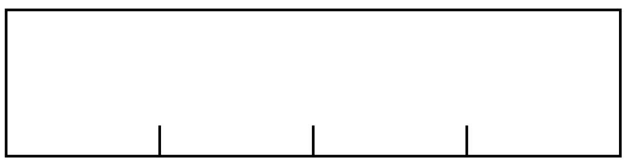
1. Which questions do you think presented the most difficulty and why?

Each of the questions is discussed on the following pages.

Fractions: Question 1

FIGURE 5: **Fractions task: Question 1**

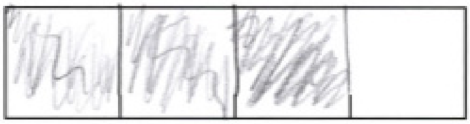
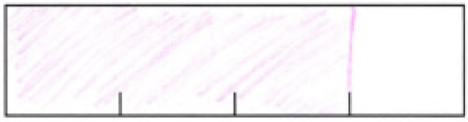
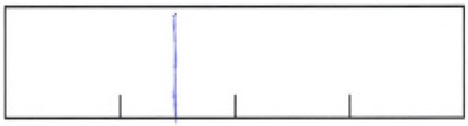
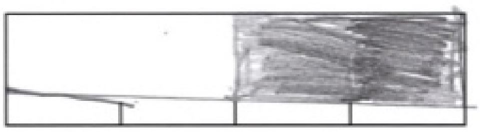
1. Shade in $\frac{3}{4}$ of this oblong.



What is the question about?

Question 1 required the students to use an area model to represent ¾. In an area model, the focus is on how the area of the shaded part compares to the area of the whole shape. In this question, the area model provides some lines to help students identify the quarters they need. Table 10 shows some of the students' responses to this question.

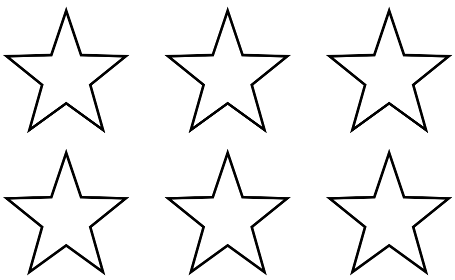
TABLE 10: **Student responses to Question 1**

 	Here the students have clearly recognised that the given lines indicate quarters. One student has continued each of the lines to create three enclosed quarters, while the other has imagined the creation of quarters and drawn a boundary at the third line. Both students have accurately shaded in 3 of the 4 parts.
	This Year 8 student has drawn a line halfway between the first and second lines. Is it possible that they saw the lines as representing halves and have marked off where ¾ would be positioned?
	This student has shaded in part of the bar. What might they be trying to show?

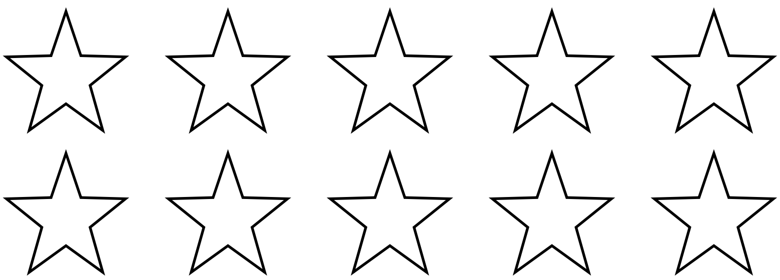
Fractions: Questions 2 and 5

FIGURE 6: **Fractions task: Questions 2 and 5**

2. Circle $\frac{1}{3}$ of this set of stars.



5. Circle $\frac{2}{5}$ of this set of stars.



What are these questions about?

Questions 2 and 5 required students to use a set model to represent a fraction. In a set model, the whole is made up of a set of objects and a subset of the whole makes up the fractional part. Note, that for both these questions the number of parts in the denominator does not match the number of stars shown. This means the students must recognise how to break up the set of stars into appropriately sized subsets.

TABLE 11: **Student responses to Question 2**

		These students have recognised that showing ⅓ involves dividing the 6 stars into 3 groups of 2 and circling 1 of the groups. It is interesting to note the different visual groupings of ⅓—vertical and horizontal. The horizontal grouping might suggest automatic application of multiplication knowledge (e.g., *I know 2 is ⅓ of 6).*
		These students' responses might suggest that the students see ⅓ as 2 separate whole numbers. They may have focused on either the numerator or the denominator or even combined them to get 4. The identification of 1 star as ⅓ of 6 may draw on the student's experiences of identifying 1 as ⅓ of 3. Perhaps they have regarded the 2 rows of 3 stars as 2 separate wholes.
		

Fractions: Questions 3 and 4

FIGURE 7: **Fractions task: Questions 3 and 4**

3. Draw a diagram to show $1\frac{1}{4}$.

4. Mark where $1\frac{1}{2}$ is on this number line.

What are these questions about?

Questions 3 and 4 required students to work with mixed fractions (fractions larger than 1 whole). Working with mixed fractions relies on understanding that they combine a whole number with a fraction. The whole number part can be turned into a fraction by 'breaking' it into the same size parts as the fraction part. For instance, 1¼ can be thought of as 4 quarters and 1 more quarter—a total of 5 quarters (⁵⁄₄).

For Question 3, students were able to choose how they would represent the fraction. Table 12 shows some of the ways students responded.

TABLE 12: **Student responses to Question 3**

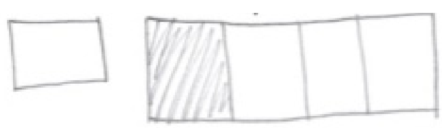
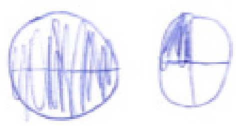
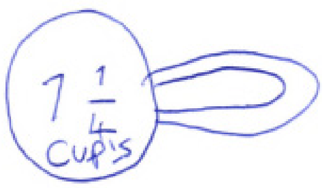
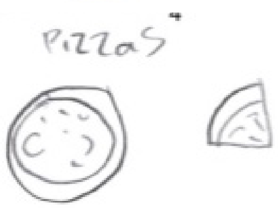
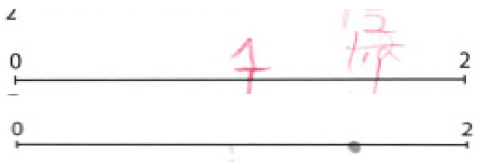
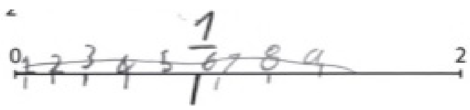
	This student attempted to use a set model. They appear to be emphasising that one of the four circles is separate or special. Perhaps they think the 1¼ means that 1 of the quarters is separate to the other 3 quarters.
	This student used an area model based on rectangles. They appear to understand that 1¼ involves 1 whole and 1 more part of a whole. However, the size of the whole is not proportional to the size of the quarters.
	This student has used an area model based on a circle. Their model clearly shows 1 whole and 1 more quarter.
 	These examples suggest prior, contextually rich experiences of exploring fractions. The first example draws on a real context for fractions and might indicate an emerging concept of 1 and ¼. The second example also draws on a real context and shows a more developed understanding of how to represent 1¼.

Table 13 shows some of the ways students responded to Question 4, which required students to use a number line. A number line is an example of a linear model that represents fractions using lengths. In a number line, 1 whole is represented by a length of 1. For this question, an important part of locating 1½ on the number line is first working out where '1' is located.

TABLE 13: **Student responses to Question 4**

	The first example clearly indicates partitioning of 1 whole and 1 half (represented by 1.5), whilst the second example shows some evidence of partitioning (although the partitions are unlabelled).
	It is possible that the student has made a guess or focussed on locating ½. There is no label or partitioning to indicate the student's thinking. Other common incorrect markings were seen at 1, ¼, ½, ¾, and 1¾.
	The shading might show that the student has tried to use the number line to create an area model. It might suggest a lack of experience working with number lines.
	This example indicates the student has some understanding of a number line. They have attempted to number the line starting from 0. However, at some point, they have recognised the halfway mark between 0 and 2 as being 1 and crossed out their original numbering. They have not attempted to show where 1½ might be positioned.

Fractions: Question 6

FIGURE 8: **Fractions task: Question 6**

6. What is $\frac{3}{4}$ as a decimal?

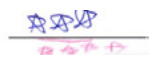
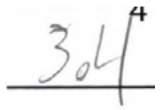
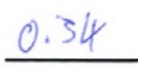
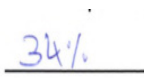
____

What is this question about?

Converting a fraction to a decimal involves understanding the place value system that underpins decimal numbers. To convert ¾ into a decimal involves recognising that ¾ can be rewritten as $\frac{75}{100}$, which can be broken down into 7 tenths and 5 hundredths. In turn, this can be rewritten as 0.75.

It is very likely that many successful students know that ¾ can be represented by 0.75 as a basic fact or benchmark. They may not have a strong conceptual understanding of decimals, however.

TABLE 14: **Student responses to Question 6**

			These responses by Year 4 and Year 8 students might suggest limited awareness of decimals. They might also suggest a high use of area and set models to represent fractions.
			Some of these students have recognised what it means to create a decimal, whilst others have applied whole number thinking or created a percentage.
			

How did the students do?

Table 15 shows the percentage of students who answered each question correctly. Does the order of difficulty agree with the estimates you made at the start of this section?

TABLE 15: **Student responses to question 1 of the fractions task**

Question	Response	Year 4 (%)	Year 8 (%)
1	Shaded ¾ of the bar.	74	94
2	Shaded ⅓ of 6 stars	32	76
3	Drew a correct diagram of 1¼	13	59
4	Located 1½ on a number line	13	63
5	Shaded ⅖ of 10 stars	21	64
6	Identified 0.75 as the decimal equivalent	2	39

4. Coloured Counters

The Coloured Counters task was completed independently by Year 4 and Year 8 students.

FIGURE 9: **The Coloured Counters task**

Coloured Counters

There are some coloured counters in a bowl.

$\frac{1}{4}$ are black.

$\frac{1}{5}$ are green.

1. a) Are there **more black** or **more green** counters in the bowl?

More Black ☐ More Green ☐ *(Tick response)*

b) Show how you got your answer. You can use words and pictures.

Questions

1. About what percentage of students do you think would answer 'More Black' at each year level?

Year 4 ☐ %

Year 8 ☐ %

2. Why might a student argue that there will be more green counters?

What is the task about?

The Coloured Counters task requires students to compare and order unit fractions (i.e., fractions with a numerator of 1). The context involves an unidentified number of objects (coloured counters in a bowl). This task also requires students to show how they got their answer with words and/or pictures.

Comparing and ordering unit fractions relies on recognising that the numerator and denominator work together to define the size of a fraction. Without this understanding, students will sometimes attempt to use what is called 'whole number thinking'. Whole number thinking refers to an overgeneralised attempt to use knowledge and strategies that work for whole numbers to problems that involve fractions. For this question, using whole number thinking might mean arguing that ⅕ of the group of counters is larger than ¼, because 5 is a larger number than 4.

How did the students do?

Table 16 shows how the students did on the Coloured Counters task, by year level.

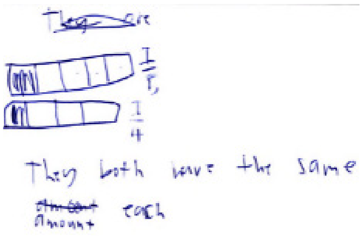
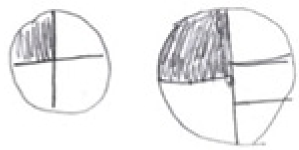
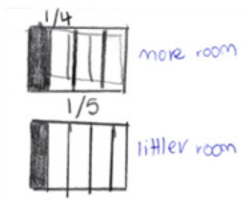
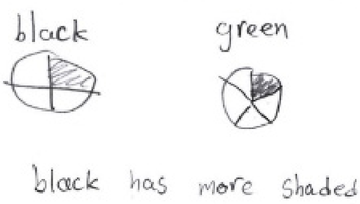
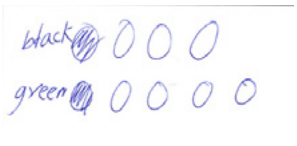
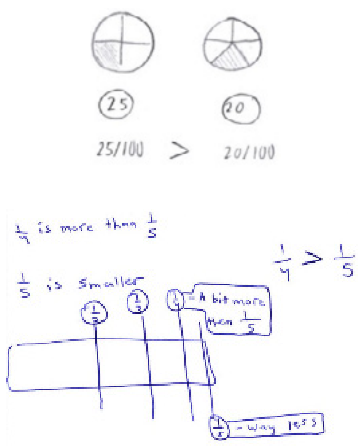
TABLE 16: **Student responses to the coloured counters task**

Response	Year 4 (%)	Year 8 (%)
Correctly identified ¼ as the larger fraction	38	77

How did students answer the question?

Table 17 shows some of the common responses given by Year 4 and Year 8 students.

TABLE 17: **Student responses to the coloured counters task**

		These responses used area models to represent and compare ¼ and ⅕. Notice that the students have struggled to divide the shapes into equal sized parts or to make each whole the same size. Experience with partitioning shapes as well as strong fine motor skills are needed here. Note that the difficulty of fraction comparison decreases as the distance between given fractions increases. It is also more difficult to partition a whole into an odd number of parts.
		These students have also used area models to compare the fractions. Compared to the examples above, the pictures show more control in representing different fractions.
		These responses attempted to use set models (i.e., a set of objects partitioned into groups) to represent and compare ¼ and ⅕.
		These students have been able to represent fractions in several different ways (i.e., as a visual model, percentages, decimals, and equivalent fractions). In turn, this indicates deep, model-rich understanding of fractions

5. Closest to $\frac{3}{5}$

'Closest to $\frac{3}{5}$' was a task presented in a multiple-choice format and followed up with an opportunity for students to explain their thinking. The task was completed by students in Year 8 and was done independently.

FIGURE 10: **The Closest to $\frac{3}{5}$ task**

Closest to $\frac{3}{5}$

1. a) Which of these is closest to $\frac{3}{5}$. ***Circle*** the answer.

   A. 0

   B. $\frac{1}{2}$

   C. 1

   D. 3

   E. 5

   F. 8

   b) Explain why you think this. You can use words and pictures.

Questions

1. Which options in question 1 will be the most popular and why?

What is this task about?

To complete this task, students need to understand that fractions represent numbers that can be compared with other numbers. By recognising that $\frac{3}{5}$ represents a number between 0 and 1, students become aware that answers D, E, and F are not possibilities. Students who understand the meaning of $\frac{3}{5}$ as a whole split into 5 pieces, of which 3 pieces are selected, can also draw on their understanding of 3 being a bit more than half of 5.

To complete the tasks, students can also draw on their understanding of percentages and decimals. For instance, $\frac{1}{5}$ is 20% or 0.2, meaning $\frac{3}{5}$ can be equated with 60% or 0.6. which is a little bigger than 0.5 or $\frac{1}{2}$.

When students struggle to understand that $\frac{3}{5}$ represents one number, they might focus on applying whole number thinking by focusing, for instance, on the denominator or numerator or both. This can make 3, 5, and 8 (3+5) look like reasonable answers.

How did the students do?

Table 18 shows the percentage of students who chose each of the options for the Closest to $\frac{3}{5}$ task. The most popular answer was $\frac{1}{2}$ (61%). Nearly all the students who did not select $\frac{1}{2}$, chose either 3, 5, or 8 as their answer.

TABLE 18: **Percentage of students selecting each option for the Closest to $\frac{3}{5}$ task**

Option	Year 8 (%)
A. 0	1
B. $\frac{1}{2}$	58
C. 1	8
D. 3	19
E. 5	10
F. 8	4

How did the students explain their answers?

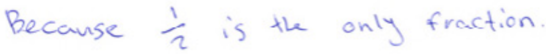
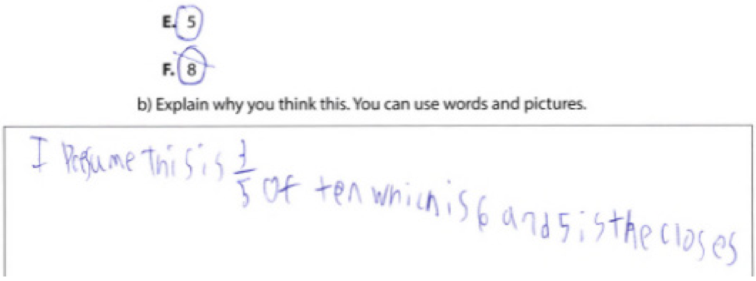
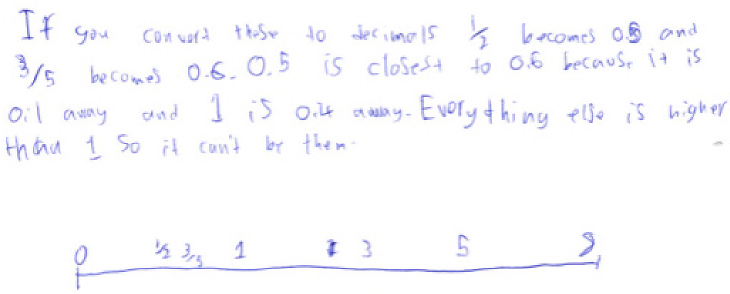
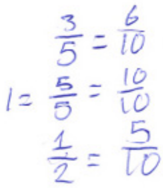
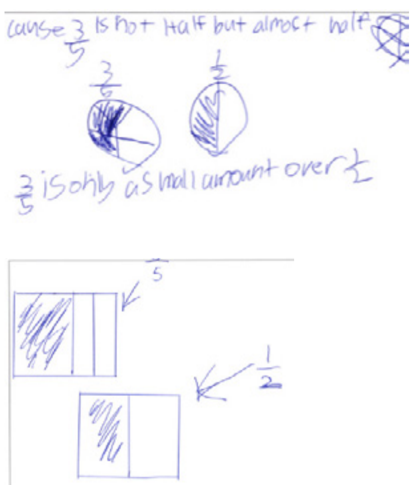
The students' explanations were also coded using a rubric (see Table 19). Although 58% of students had selected the correct answer, a much smaller proportion provided a sound justification.

TABLE 19: **Student responses to the closest to ⅗ task**

Response	Year 8 (%)
Demonstrated little understanding of how to relate ⅗ to other numbers.	56
Gave some correct reasoning for their answer.	6
Accurately related ⅗ to other numbers and provided sound justification for their answer.	38

Table 20 below shows some of the common explanations given by Year 8 students.

TABLE 20: **Examples of student responses to the 'Closest to ⅗ task'**

	This student has argued that because ½ is the only fraction provided as an option, it must be the best answer. This student has not recognised fractions can be ordered alongside whole numbers.
	This student wants the fraction to operate on another number (they have chosen 10). This leads them to select 5.
	This student has used their understanding of decimals and number lines to clearly demonstrate and justify why ½ is the closest number to ⅗.
	This student has applied their knowledge of equivalent fractions to show how ½ relates to ⅗ and to 1.
	These students have used visual diagrams to compare the size of the fractional parts in ½ and ⅗.

Final thoughts

This booklet started by looking at the problem ½ + ¼ = ?. As part of the 2022 NMSSA study, we interviewed around 300 students at each of Year 4 and Year 8 about the problem. The interview was in three stages. First, we presented the problem in its original symbolic form; then we checked the students were able to read the question and supported them to read it where needed; finally, we asked it again, but this time in a context: You have half a pizza, and a friend gives you another quarter of a pizza—how much do you have now?

Table 21 shows the percentage of students who were able to come up with the correct answer at each stage. As can be seen, the total percentage correct after Stage 3 was greater than the percentage correct after the first stage.[5]

TABLE 21: **Percentage correct at each stage, by year level**

	Year 4 (%)	Year 8 (%)
Stage 1 (original form)	9	39
Stage 2 (after reading check)	7	7
Stage 3 (in a context)	6	18
Total percentage correct	22	64

The results from the interview indicate that many students did have some knowledge they could use to work with the fraction concepts that underpin this question. Sometimes, they required support to activate themselves as problem solvers so they could work their way to a solution. There was also a sizeable proportion (36%) who found it hard to make sense of the problem even after Stage 3.

We hope that this booklet provokes some responses in classrooms that lead to more students gaining the knowledge, skills, confidence, and problem-solving dispositions they need to be successful with fraction questions like this one. In the appendix that follows, we have listed some of the key ideas about fractions covered in this booklet, along with some teaching activities and resources that might be useful when meeting this challenge.

5 At Stage 1, 19% of Year 4 students and 33% of Year 8 students wrote ⅖ as the answer. These students have applied whole number thinking and lost the meaning of the equation.

Appendix
Some key fraction ideas and teaching suggestions

A fraction signifies a relationship between a whole and the number of equivalent parts it is divided into.

Ask students to draw a simple shape such as a circle, rectangle, or star. Ask them to shade a common unit fraction of their shape, e.g., $\frac{1}{4}$. Notice how they partition the shape. Are they partitioned into equal parts? Ask if there are different ways of partitioning the shape to show the unit fraction? How can they show the parts are of equal size?

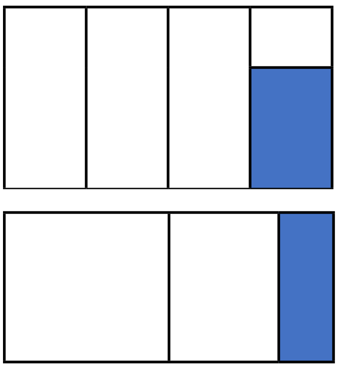
Link what they have drawn to fraction symbols. Discuss which part of the symbol represents the number of shaded parts (numerator) and which part represents the number of equal parts that make up the whole (denominator).

Explore contexts where students come across fractions in everyday life. Collect examples of fractions, e.g., in recipes, adverts, articles, measuring cups etc.

Fractions can be represented with a range of models.

Provide students with the name of a fraction or its symbol. Ask students to represent the fraction in different ways. Ask them to explain how their representation works. Explore the different kinds of representations the students have created. What is similar and what is different across the representations?

Provide students with area models where they need to work out how much of the shape has been shaded (see examples below showing $\frac{2}{12}$ and $\frac{1}{6}$, respectively). Ask the students how they can prove their answers. Their method might involve drawing more lines, measuring, folding or even cutting to compare the pieces. Student can also construct their own examples.



Unit fractions (e.g., ¼) can be iterated (joined together) to construct fractions up to, and greater than, one whole.

Choose a set of unit fractions from a set of fraction strips (see the description of fraction strips below) and join them together to make other fractions. Count how many unit fractions are needed to make the new fraction. For example, students can join three ⅛s together to make ⅜, students. They can count ⅛, 2/8, ⅜ to show that it takes 3 of these units to reach ⅜. Ask them to compare the fractions they make with the one whole. How many more 1/8s will be needed to make the whole? Ask how they can tell when something is ⅛? It will take eight copies of it to make a whole.

Fractions can be compared with each other

To compare fractions we have to be aware of the size of the parts and the number of parts. When we use models to compare the size of fractions, we have to take care we construct the models accurately.

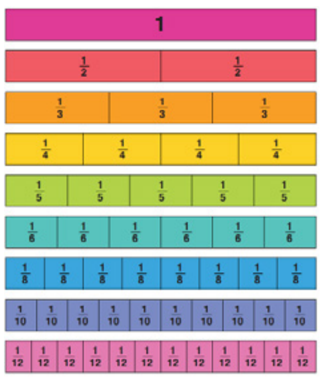
Ask students to use area models of fractions made from paper strips to represent and compare two unit fractions that are close in size (e.g., ¼ and ⅕). Notice how the students partition the strips. Are they careful to make sure each strip is partitioned accurately? Ask students what happens to the comparison if they are not? Ask them why we start with the same size paper strips to represent the whole when comparing two fractions? What happens if we don't?

Fractions can be equivalent to other fractions

Ask the students to use fraction strips to show how two fractions can represent the same quantity, For instance: Use the fraction strips to find fractions that are equivalent to ¾. What patterns can they see?

Fractions are numbers that can be located on a number line alongside whole numbers.

Use fraction strips to help students locate the position of different fractions along an empty number line. For instance, they can line up the ⅛s and draw a number line underneath starting at 0 and ending at 1. Next, they can mark and label the ⅛s on the number line. Ask where a mixed fraction like 1 ⅜ will be located on the number line.



Roll two dice and use the numbers rolled to make a fraction e.g., roll 2 and 6 to make the fraction 2/6. Ask the students to locate the fraction on a number line. Students will need to draw a line marked from 0 to 1, and then partition the line into 6 equal parts. They then need to mark 2/6 on the line.

Locate fractions on number lines marked out in different ways. For instance, show where ¼ is on the number line below.

Fractions can be represented by decimals, percentages, or other, equivalent fractions. When these symbols represent the same quantity (e.g., $\frac{1}{2}$, 50%, 0.5), they occupy the same position on a number line.

Use knowledge of benchmarks and multiplication to explore how a fraction can be represented by different symbolic forms (e.g., $\frac{4}{6}$ and $\frac{2}{3}$). Extend the fraction strips idea to include strips that show fractions, decimals and percentages, Use these along with number lines to locate, compare, and equate different, notable fraction, decimal, and percentage amounts (e.g., $\frac{1}{2}$, 50%, $\frac{5}{10}$, 0.5).

Working with fractions involves being a confident problem solver.

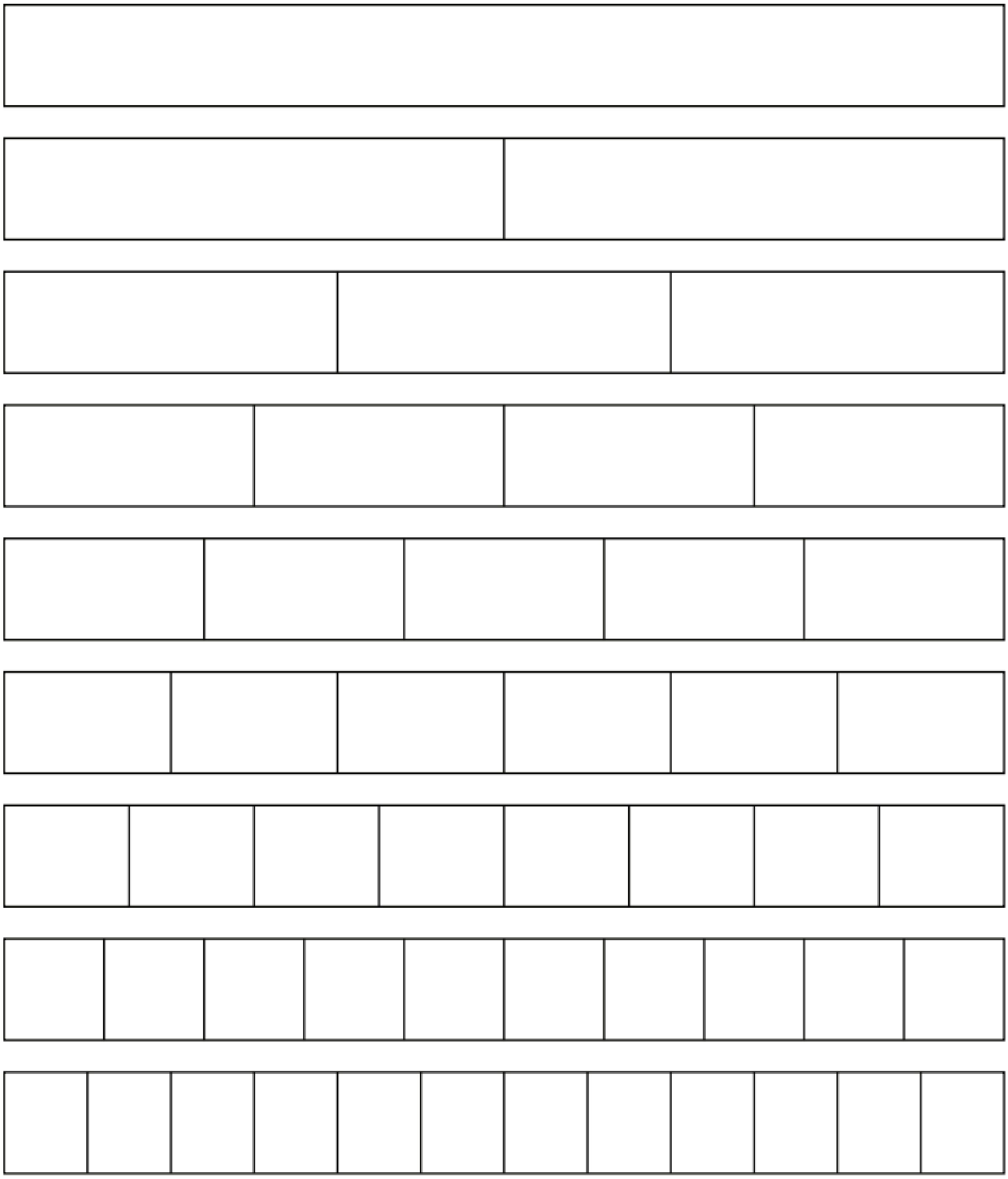
Encourage students to build, apply, and reinforce their knowledge of fractions through problem solving. Use a cycle of mathematical investigative thinking (e.g., make sense of the problem and connect to prior knowledge, choose and explore a problem-solving strategy such as 'draw a picture', apply the approach, and justify and generalise findings) to structure problem solving.

Students need lots of opportunities with fractions—learning about fractions requires time and multiple exposures to different representations and fractional meanings, including their real life applications.

Use a range of models (e.g., area, number line, strip, set), alongside relevant vocabulary (e.g., "three quarters"), and symbols (e.g., $\frac{3}{4}$) to develop strong conceptual and procedural understanding of fractions. This should involve drawing on prior knowledge of whole numbers and support understanding of fractions up to, and greater than one. Connect the learning about fractions done in the classroom to the experiences students are having in their everyday lives.

Fraction strip template

Fraction strips are rectangular pieces of coloured paper or card that students can use to learn about fractions. Students can have fun making and labelling their own fractions strips using coloured paper (use the template below which is already marked out, or give them a pre-cut set of strips that they mark or fold themselves to make the different fractions).



Some useful resources

For further teaching ideas, consider the following resources from NZ Maths (https://nzmaths.co.nz/) and the Assessment Resource Banks or ABRs (https://arbs.nzcer.org.nz/).

- The Assessment Resource Banks (ARBs): Fractional Thinking Conceptual Map
- **Equally partitioning shapes and areas:** Sharing shapes (ARBs), Cutting the cake (ARBs), Fraction shapes (ARBs), Fractional Blocks (NZ Maths), Unit Fractions (NZ Maths), More Unit Fractions (NZ Maths), Non unit Fractions (NZ Maths)
- **Identifying whether fractional parts are equal:** Equal Parts (NZ Maths)**,** Fraction Circles (NZ Maths),
- **Physically and diagrammatically constructing fractions:** Plasticene fractions (ARBs) Finding two-thirds (ARBs)
- **Ordering fractions:** Ordering improper and mixed fractions (ARBs) Ordering fractions (ARBs) Different fractions on a number line (ARBs), Ordering proper and improper fractions (NZ Maths)
- **Sharing and Partitioning Sets:** Partitioning sets (ARBs), Shading fractions of sets and shapes (ARBs), Fractions and sets (ARBs), Shaded fractions (ARBs), Fair Shares (NZ Maths),
- **Finding given fractions of a set:** Fractions of sets (ARBs), Birthday Cakes (NZ Maths)
- **Developing benchmarks:** Close to a half (ARBs), Fraction Benchmarks (NZ Maths)
- **Equivalent fractions:** Equivalent Fractions (NZ maths), Equivalent fractions with sets (NZ maths)
- **Reconstructing wholes from given fractions:** Parts and wholes (ARBs), What's the whole unit? (ARBs)

Some useful books

Petit, M. M., Laird, R. E., Ebby, C. B., & Marsden, E. L. (2003). *A focus on fractions: Bringing mathematics education research to the classroom*. Routledge.

Neagoy, M. (2017). *Unpacking fractions: Classroom-tested strategies to build students' mathematical understanding*. Association for Supervision & Curriculum Development.

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